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ON THE SOLVABILITY OF NONLINEAR EQUATIONS FOR A SYMMETRICALLY LOADED NONSHALLOW SPHERICAL DOME

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Existence of the generalized solution in the problem of equilibrium of the isotropic elastic nonshallow spherical dome with rigidly held edge and subjected to axisymmetric deformation is proved by the method presented in [1]. The topological characteristic of the problem, i.e. the vector field rotation is computed. The solvability of nonlinear equations for nonshallow shells of revolution subjected to symmetric load was investigated in [2, 3]. However dome-shaped shells were not considered there.

1. Fundamental relationships. We consider the following version of relationships of the nonlinear theory of nonshallow symmetrically loaded shells of revolution:

$$
\begin{aligned}
&T_1(\varepsilon_j) = K(\varepsilon_1 + \nu\varepsilon_2), \quad M_1 = D(\varkappa_1 + \nu\varkappa_2) \quad 1 \rightleftarrows 2 \qquad (1.1)\\
&\varepsilon_1 = vA'(AB)^{-1} + wR_1^{-1}, \quad \varepsilon_2 = v'B^{-1} + wR_2^{-1} + \psi^2 2^{-1}\\
&\varkappa_1 = -\psi A'(AB)^{-1}, \quad \varkappa_2 = -\psi' B^{-1}, \quad \psi = w'B^{-1} - vR_2^{-1}\\
&T_{12} = M_{12} = \varepsilon_{12} = \varkappa_{12} = 0\\
&K = 2hE(1-\nu^2)^{-1}, \quad D = 2h^3[3(1-\nu^2)]^{-1}E
\end{aligned}
$$

where T_i and T_{12} are tangential stresses; ε_i and ε_{12} are the tensile and shear strains, respectively; M_i and M_{12} are, respectively, the bending moment and the torque; $\varkappa_i$ and $\varkappa_{12}$ are changes of curvature R_i^{-1} of the shell middle plane s^*; v and w are, respectively, the tangential and normal displacement of the shell middle plane s^*; A^2, B^2, $2C = 0$ are coefficients of the first quadratic form of surface s^*; $E > 0$ is the Young modulus; $0 < \nu < 1/2$ is the Poisson ratio, and $2h$ is the thickness of the shell. A prime superscript denotes differentiation with respect to parameter β.

The analysis of a spherical dome is conveniently carried out in spherical coordinates un which $A = \rho \sin\beta$, $\beta \in [0, b]$, $B = R_i = \rho$, where ρ is the radius of the shell middle plane s^*. For convenience we set $\rho \equiv 1$. The substitution $v = w' - \psi$ eliminates v from all formulas. We introduce the notation

$$
e_1(w) = w' \operatorname{ctg}\beta + w, \qquad e_2(w) = w'' + w \qquad (1.2)
$$

The equation of the shell equilibrium is determined by the Lagrange principle which yields

$$\int_0^b (T_i(\varepsilon_j)\,\delta\varepsilon_i + M_i\delta\varkappa_i)\sin\beta\,d\beta = \int_0^b [F_1(\delta w' - \delta\psi) + F_2\delta w]\sin\beta\,d\beta \tag{1.3}$$

$$\delta\varkappa_i = \varkappa_i(\delta\psi), \quad \delta e_i = e_i(\delta w), \quad \delta\varepsilon_2 = \delta e_2 + \delta\varkappa_2 + \psi\delta\psi$$

where F_i are components of the external load. Since the shell edge is rigidly held,

$$\psi(b) = w(b) = w'(b) = 0 \tag{1.4}$$

The axial symmetry of the shell makes it possible to assume that

$$\psi(\beta) = -\psi(-\beta), \qquad w(\beta) = w(-\beta) \tag{1.5}$$

Let condition

$$b < \pi / 2 \tag{1.6}$$

be satisfied.

We introduce the scalar products

$$(\psi\cdot\delta\psi)_{H_1} = \int_0^b M_i\delta x_i \sin\beta\,d\beta$$

$$(w\cdot\delta w)_{H_2} = \int_0^b T_i(e_j)\,\delta e_i \sin\beta\,d\beta$$

Definition 1. The closure of the set C_1 $\{C_2\}$ of 2π-periodic functions $\psi \in C^{(1)}\{w \in C^{(2)}\}$ on $[0, b]$, which satisfies conditions (1.4) and (1.5) in the related norm, is called Hilbert space H_1 $\{H_2\}$

$$\|\psi\|^2_{H_1} = \int_0^b M_i\varkappa_i \sin\beta\,d\beta = D\int_0^b [\psi'^2 + (\psi\operatorname{ctg}\beta)^2 + 2\nu\psi'\psi\operatorname{ctg}\beta]\sin\beta\,d\beta$$

$$\|w\|^2_{H_2} = \int_0^b T_i(e_j)\,e_i\sin\beta\,d\beta = K\int_0^b [(w'' + w)^2 + (w'\operatorname{ctg}\beta + w)^2 +$$

$$2\nu(w'' + w)(w'\operatorname{ctg}\beta + w)]\sin\beta\,d\beta$$

Lemma. If function $\psi \in H_1$ $\{w \in H_2\}$, then $\psi s_p \in C^{(0)}$ $\{w's_p,\ w \in C^{0}\}$ on $[0, b]$, $s_p = (\sin\beta)^{1/p}$ and $p > 1$. The weak convergence of $\psi_n \to \psi_0$ in H_1 $\{w_n \to w_0$ in $H_2\}$ for $n \to \infty$ implies a strong convergence $\psi_n s_p \Rightarrow \psi_0 s_p$ $\{w_n \Rightarrow w_0,\ w_n's_p \Rightarrow w_0's_p\}$ in $C^{(0)}_{[0,b]}$.

Proof. Since by condition (1.5) $\psi(0) = 0$, hence

$$f(\beta) = \psi(\beta)\,s_p(\beta) = \int_0^\beta (\psi(\alpha)\,s_p(\alpha))'\,d\alpha = \int_0^\beta [\psi'(\alpha) + \tag{1.7}$$

$$p^{-1}\psi(\alpha)\operatorname{ctg}\alpha]\,(\sin\alpha)^{1/p}\,d\alpha = \int_0^\beta (\sin\alpha)^{1/p-1/2} M(p,\alpha)\,d\alpha$$

$$f(\beta + t) - f(\beta) = \int_\beta^{\beta+t} (\sin\alpha)^{1/p-1/2} M(p,\alpha)\,d\alpha$$

$$M(p,\alpha) = [\varphi'(\alpha) + p^{-1}\psi(\alpha)\operatorname{ctg}\alpha]\,(\sin\alpha)^{1/2}$$

Using the Hölder inequality, from (1.7) we obtain

$$|f(\beta)| \leqslant \left(\int_0^b (\sin\alpha)^{2/p-1}\,d\alpha\right)^{1/2} L, \qquad p > 1 \tag{1.8}$$

$$|f(\beta+t)-f(\beta)| \leqslant \left|\int_0^t (\sin(\beta+\alpha))^{2/p-1}\,d\alpha\right)^{1/2} L \leqslant \eta(t)\left(\int_0^\beta M(p,\beta)^2\,d\beta\right)^{1/2}$$

$$\eta(t)=\begin{cases} t & 1<p\leqslant 2 \\ \left(\int_0^t (\sin\alpha)^{2/p-1}\,d\alpha\right)^{1/2}, & 2<p,\ \beta+t\leqslant \pi/2 \end{cases}$$

$$L\equiv\left(\int_0^\beta M(p,\beta)^2\,d\beta\right)^{1/2}\leqslant m_1\|\psi\|_{H_1}$$

where m_k are constants. The validity of the lemma for functions $\psi\in H_1$ follows from (1.8) and the Arzelà theorem.

Using conditions (1.4)−(1.6) it is possible to show that the relationships

$$0<m_2\leqslant\|w'\|_{H_1}\|w\|_{H_2}^{-1}\leqslant m_3 \tag{1.9}$$

$$\|w\|_{C^{(0)}_{[0,b]}}\leqslant\|w'(\sin\beta)^{1/4}\|_{C^{(0)}_{[0,b]}}\int_0^b(\sin\beta)^{-1/4}\,d\beta$$

are satisfied. It follows from (1.9) that the lemma is also valid for functions $w\in H_2$.

2. Statement and solvability of the problem. As in [1], we introduce the concept of the generalized solution.

Definition 2. We call the pair of functions $\psi\in H_1$ and $w\in H_2$ the generalized axisymmetric solution of the problem of equilibrium of an elastic nonshallow spherical dome with a rigidly fixed edge subjected to an axisymmetric load. The pair of functions must satisfy Eq. (1.3) for any pair of functions $\sigma\psi\in H_1$ and $\sigma w\in H_2$.

All terms of Eq. (1.3) have a meaning, if

$$F_1\in H_{-1},\qquad F_2\in H_{-2} \tag{2.1}$$

where $H_{-1}\,\{H_{-2}\}$ is an adjoint space of $H_1\{H_2\}$.

Repeating the reasoning of [2] and taking into account condition (1.6) and the lemma, it is possible to show that Eq. (1.3) reduces to the operator equation $\psi=G\psi$, where G is an absolutely continuous operator acting in H_1. On spheres $T(R,0)=\{\psi\in H_1:\|\psi\|_{H_1}=R\}$ of a fairly large radius R the rotation of the absolutely continuous vector field $I-G$ (I is a unit vector) in H_1 is equal +1, i.e. [4]

$$\gamma(I-G;\ T(R,0))=+1 \tag{2.2}$$

The following theorem is based on the Leray-Shauder principle [4].

Theorem. If conditions (1.6) and (2.1) are satisfied, there exists at least one generalized axisymmetric solution, in the meaning of Definition 2, of the problem of equilibrium of an isotropic elastic nonshallow spherical dome with rigidly fixed edge subjected to axisymmetric load.

Note. (1) The theorem is valid for various dome-shaped symmetrically loaded shells of revolution, whose coefficients of the first quadratic form in (1.1) satisfy the following conditions:

a) $A(\beta)$ monotonically increases on $[0,b]$ which is the considered region of variation of parameter β;

b) $\int_0^\beta A(\beta)^{-1+\varepsilon}\,d\beta<\infty$ ($\varepsilon>0$ is the required reasonably small number;

c) $0 < m_4 \leqslant B(\beta)$ and $R_i(\beta) \leqslant m_5$ on $[0, b]$.

Dome-shaped shells whose middle surface is a part of an ellipsoid, paraboloid, two-sheet hyperboloid, and other surfaces of revolution.

2) According to [4] condition (2.2) guarantees the convergence of projective methods.

3) Similar results can be obtained in the case of a shallow symmetrically loaded spherical dome and other shallow symmetrically loaded surfaces of revolution ($\psi = w' B^{-1}$ in (1.1)), if conditions (a), (b), (c) are satisfied.

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CAUCHY PROBLEM FOR A VISCOELASTIC TRANSVERSELY ISOTROPIC MEDIUM

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We solve a Cauchy problem for a viscoelastic transversely isotropic medium. Generalizing the method of separation of variables for certain classes of static problems treated in [1], and using this method, we reduce the system of integro-partial differential equations to a system of ordinary differential equations in the time coordinate. Solving the latter system by the method of averaging [2, 3], we obtain explicit formulas characterizing the propagation of waves in a viscoelastic transversely isotropic medium.

Using the relationship between stress and strain for the medium in question [4] and identifying the regular part of the relaxation kernels, we write the system of equations for a viscoelastic transversely isotropic medium in cylindrical coordinates as follows:

$$\begin{gathered}
(c_{66} - c_{66}^*)\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2} \pm \frac{2i}{r^2}\frac{\partial}{\partial \varphi} - \frac{1}{r^2}\right) W_{1,2} + \\
(c_{55} - c_{55}^*)\frac{\partial^2 W_{1,2}}{\partial z^2} + \left(\frac{c_{11} + c_{12}}{4} - \frac{c_{11}^* + c_{12}^*}{4}\right)\left(\frac{\partial}{\partial r} \pm \frac{i}{r}\frac{\partial}{\partial \varphi}\right) \times \\
\left[\left(\frac{\partial}{\partial r} - \frac{i}{r}\frac{\partial}{\partial \varphi} + \frac{1}{r}\right) W_1 + \left(\frac{\partial}{\partial r} + \frac{i}{r}\frac{\partial}{\partial \varphi} + \frac{1}{r}\right) W_2\right] + \\
[(c_{13} + c_{55}) - (c_{13}^* + c_{55}^*)]\left(\frac{\partial}{\partial r} \pm \frac{i}{r}\frac{\partial}{\partial \varphi}\right)\frac{\partial W_3}{\partial z} = \rho\frac{\partial^2 W_{1,2}}{\partial t^2}
\end{gathered} \tag{1}$$